An optimisation-based traffic regulator for metro lines

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Abstract

This paper presents a new traffic regulation model for metro lines based on the optimisation of a cost function along a time horizon. The resulting control actions modify nominal running times and dwell times to compensate timetable and headway deviations. The proposed quadratic programming model is suitable to include the main operation constraints efficiently: minimum interval, limits in the control actions and the typical operation criterion of preventing the actuation of signalling systems between platforms. Quadratic programming models are computationally efficient, and this allows the execution of the control loop every few seconds, and the management of long time horizons. In consequence, regulation performance and stability is improved, and the tuning of regulation parameters according to the operation requirements is simplified. The paper describes the simulation results of the proposed regulation model applied to a realistic metro line. A traffic perturbation model has been considered in order to analyse the stability of the line with different configuration parameters of the controller.
Keywords: railway traffic control, optimisation, predictive control.

1 Introduction

It is well known that metro lines are unstable, because train delays are propagated to the following trains by the signalling systems when the minimum headway between successive trains is reached [1], and delays increase at each station due to the accumulation of passengers [2]. Traffic regulation strategies are then necessary to prevent the traffic degradation and to observe the quality requirements.

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Regulation strategies try to recover train delays by certain time margins included both in the nominal dwell times at platforms and in the nominal runtimes. Small delays or early departures could be compensated by the train using its own time margins, being the control action local to each train. With larger delays, trains need several stations to compensate deviations, and a transient period is established to reach the nominal operation. The regulation strategy tries to minimise mainly timetable and/or headway time deviations during the transient.

Typically, when a train is delayed, trains ahead are first delayed as well to reduce the headway, and afterwards they are forced to speed up towards nominal operation. Trailing trains are regulated in a similar way, where trains are initially held to avoid the actuation of protection systems. This strategy was initially proposed in [3] for implementation on the Paris Metro (RATP).

In metro lines operated according to a timetable, the main goal of regulation is a full timetable recovery, but the regularity of headways during the transient is usually considered to avoid the accumulation of passengers. On the other hand, in metro lines operated according to a published headway (as Metro de Madrid), the two quantifiable performance indicators in use are headway regularity and commercial speed. In order to keep commercial speed, delays with respect to a reference timetable must be completely recovered. In conclusion a compromise between timetable and headway regulation must be reached.

In the presence of large delays, the magnitude of time deviations from nominal operation could be unacceptable, and the accumulation of passengers could prevent time recovery. In these cases a new delayed timetable should be established as the regulation reference.

Different regulation strategies have been proposed, based on the minimisation of a cost function containing the 'system' performance along an optimisation horizon [2, 4]. The performance criteria are basically the delays referred to the timetable and headway deviations. The magnitude of control actions is also penalised in the cost function, so that nominal set point stands with null control actions. In [2], a simplified traffic model is considered, where bounds on control actions and minimum interval between trains are not included. This approach allows the theoretical stability analysis of metro lines but present limitations on its applicability. In [4] a predictive controller is proposed, including bounded control actions and minimum interval constraints, and it is solved by non-linear programming techniques. The main drawback of this approach is that the optimisation horizon has to be limited to 5-6 stations, due to the real time computation load.

Other predictive strategies try to minimise a cost function not based on the 'system' performance but on the passengers disturbance due to train delays. In [5] the cost function is independent on the timetable, and it is minimised using non-linear programming. A similar approach is found in [6] to schedule commercial timetables, minimising the passengers disturbance along a planning horizon (typically a whole operation day), using linear programming.

There are regulation strategies based on heuristic criteria. In [1] a heuristic predictive controller computes the complete trajectory of each train until delays are recovered. A heuristic feedback controller is proposed in **[7],** where the control law depends on the last measured delay of the train to be controlled, the delays of the train ahead and the following train.

This paper is focused on a predictive control approach that optimises a system performance cost function. The optimisation problem is formulated as a convex quadratic programming model. The cost function is based on the one used by [2] and [4]. However, this model considers bounds for control actions, the minimum interval due to signalling systems, and it is able to manage long-term optimisation horizons in real time until delays are completely recovered.

2 Optimisation model

Traffic is modelled as a set of N trains running through a cyclic sequence of M platforms, where each train has to stop because of the passengers. Metro lines with terminal stations verify this model too (Figure 1), where each station is composed by two platforms, one for each sense. Return points are modelled as a terminal commercial platform (one-platform return, Terminal A in Figure 1), or as a terminal technical platform (two commercial platform return, Terminal B in Figure 1).

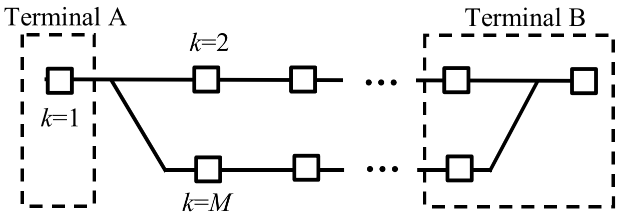

Figure 1.

Given a reference timetable Td_k^i for the departure time of each train i at each platform k, and a nominal stop time S_k at each platform k, the arrival reference timetable Ta_k^i stands (1). The nominal interval Hn between consecutive trains (where train i follows train i-1) is supposed to be constant and verifies (2) for all k, i. Given the nominal runtime R_k from platform k to platform k+1 the reference arrival time Ta^i_{k+1} at platform k+1 can be written as (3). To obtain periodic circulation of trains on the loop line the equation (4) must be verified. During the circulation of trains, departure td_k^i and arrival ta_k^i times are measured. Thus, different time deviations from nominal operation can be defined. Time deviation from nominal departure of train i at platform k is defined as (5). Time deviation from nominal arrival of train i at platform k is defined as (6). Time deviation from nominal headway of train i leaving platform k is defined as (7).

$$Td_k^i = Ta_k^i + S_k \tag{1}$$
$$Hn = Td_k^i - Td_k^{i-1} \tag{2}$$

$$Ta^i_{k+1} = Td_k^i + R_k \tag{3}$$

$$N \cdot Hn = \sum_k (S_k + R_k) \tag{4}$$

$$Xd_k^i = td_k^i - Td_k^i \tag{5}$$

$$Xa_k^i = ta_k^i - Ta_k^i \tag{6}$$

$$Y_k^i = (\, td_k^i - td_k^{i-1}\,) - H = Xd_k^i - Xd_k^{i-1} \tag{7}$$

If the departure-arrival interval between a train i and the preceding one i-1 is greater than the minimum interval *Imin*, then the run time of train i from platform k to k+1 verifies:

$$ta_{k+1}^i - td_k^i = R_k + ur_k^i \tag{8}$$

where ur_k^i is the train i control action that modifies its nominal run time from platform k to k+1. These control actions are bounded, that is:

$$URmin_k \leq ur_k^i \leq URmax_k \tag{9}$$

and the previous limits are related to the minimum and maximum run times: (R_k + $URmax_k$) is the run time from k to k+1 when the slowest coasting is applied, and (R_k + $URmin_k$) is the minimum run time, when ‘flat out’ is applied.

Subtracting (3) to the previous run-time constraint (8), a time deviation expression is obtained:

$$Xa_{k+1}^i - Xd_k^i = ur_k^i \tag{10}$$

This regulation strategy tries to avoid the actuation of signalling systems between platforms. When it is predicted that the signalling system may affect a train, the regulation system holds the train at the station as needed. As a consequence, the runtimes presented in equation (8) are not increased due to the signalling system. This regulation constraint is also considered in [1], where the successive trains are separated by the minimum headway of the line (more restrictive than the minimum headway of the platform)

Runtimes can be easily predicted along the transient, because it is not necessary to model the effects of signalling. Undisturbed runtimes are quite stable if driving is controlled by an Automatic Train Operation system.

In metro-type lines, the arrival time of a train i to a platform k is restricted by the departure time from k of the preceding train (i-1):

$$ta_k^i - td_k^{i-1} \geq Imin_k \tag{11}$$

where $Imin_k$ is the minimum dynamical interval at platform k arrival (departure-arrival interval). Notice that $Imin_k$ does not contain the nominal stop time at platform k. Subtracting the nominal time equations (1) and (2) to (11), the following deviation time expression is obtained:

$$Xa_k^i - Xd_k^{i-1} \geq Imin_k + S_k - H \tag{12}$$

The proposed stop-time model assumes that a deviation in the arrival-departure headway increases the departure deviation, due to the additional accumulation of passengers. This relation is considered linear (constant α_k):

$$Xd_k^i = Xa_k^i + \alpha_k \cdot [\, Xa_k^i - Xd_k^{i\text{-}1} \,] + up_k^i \tag{13}$$

The control action at platforms up_k^i shall be computed in order to improve the regularity performance index, and also to prevent early departures and signalling disturbances during the next runtime. A train that has already departed could be affected by the signalling system, due to a subsequent delay of the preceding train. In this case, the runtime constraint (10) of the affected train has to be suppressed, and the train will be supposed to arrive at the minimum interval (equation (12) as equality).

Notice that this traffic model is linear and it includes linear inequality constraints. Other approaches, like [4], use a non-linear simulation model to consider an equivalent system.

Cost function

The predictive traffic controller proposed in this paper minimises a system cost function including regularity criteria during the transient and also the magnitude of the control actions:

$$\begin{aligned} J' = {} & p \cdot \sum_{i,k} (Xd_k^i)^2 + q \cdot \sum_{i,k} (Xd_k^i - Xd_k^{i-1})^2 + \\ & + ap \cdot \sum_{i,k} (ur\_p_k^i)^2 + an \cdot \sum_{i,k} (ur\_n_k^i)^2 + \\ & + bp \cdot \sum_{i,k} (up\_p_k^i)^2 + bn \cdot \sum_{i,k} (up\_n_k^i)^2 \end{aligned} \tag{14}$$

for each train i: $1 \leq i \leq N$, for each platform k: $k_{0i} \leq k \leq k_{0i} + L$, where k_{oi} is the next departure platform of train i, and L is the number of platforms to be included in the optimisation horizon.

The constant p weights the timetable deviation term in the cost function J. The constant q weights the headway deviation. The variables $ur\_p_k^i$ are the runtime control actions slower than nominal speed, $ur\_n_k^i$ are the runtime control actions faster than nominal speed, $up\_p_k^i$ are the stop-time control actions that hold the train at the station, and $up\_n_k^i$ are the control actions that reduce the stop time. All the control actions hold positive values.

It is possible to increase the penalty weighting on control actions that increase the stop times and runtimes (*ap*, *bp*) compared to the penalty weighting that decrease them (*an*, *bn*). Furthermore, the energy consumption can also be independently penalised increasing the weight *an* of the negative runtime control actions $ur\_n_k^i$.

This model could be easily enhanced with particular weights for each platform, in order to stress the relative importance of local regularity and control actions (terminal stations, connections to other lines, etc.).

If the line is operated according to a published timetable, the value of p must be large enough compared to the value of the other weights, in order to allow the total delay compensation. On the other hand, if the line is operated according to a published headway, the value of p could be theoretically null, as analysed in [2] for the unconstrained traffic model. However, the solution reaches headway regularity, but delays are not totally compensated. That is, the commercial speed is degraded even when it is not necessary. It is thus convenient to introduce delay compensation in the model as a soft constraint, assigning to p a non-zero value large enough to observe the commercial speed goal.

Furthermore, the values of regularity weights p and q have to be large enough to make the system stable, as analysed in [2] with the unconstrained one-step problem. Also, it is well known that the stability of predictive controllers improves when the horizon L is enlarged [8]. The proposed method is very efficient solving long-term problems, so the parameter L can be easily adjusted to obtain the control stability.

3 Real-time control procedure

At each control cycle, of about 5 seconds, control actions are updated in real time, solving the optimisation problem previously defined by (9), (10), (12)–(14). Each time the model is solved, the control actions of every train for the whole time horizon are obtained. Then, the regulation system considers the first control action for each train (next control action required). Notice that this control action for each train could be updated at each control cycle before it is sent to the train, thus only the most recently computed control action is effectively applied.

The proposed predictive controller makes use of a fixed-length optimisation window with receding horizon, or sliding window for short, typical in this kind of regulation systems as [5]. Due to the arrival of non-regular disturbances at unpredictable moments, it is convenient to perform the optimisation over a fixed time interval into the future, but also to repeat the optimisation on a short cycle. The length of the window (optimisation horizon) is measured in number of platforms. A maximum value of the delay is assumed such that for greater delays the reference timetable is no longer acceptable, and the delay will not be completely compensated. A large value L_0 of the length of the optimisation horizon can be found for the maximum delay, such that delays are completely compensated and an increment of the horizon does not vary the optimisation solution significantly, that is:

$$J(L_0) - J(L_0+1) < \varepsilon \qquad (15)$$

where $J(L_0)$ and $J(L_0+1)$ are the optimisation solutions obtained for horizon lengths L_0 and L_0+1.

As previously mentioned, the optimisation with large horizons is adequate to improve stability, but if the regularity weights p and q were small compared to control action weights, the control system could be unable to compensate delays, or even to make the system stable.

The proposed optimisation model is a convex quadratic programming one, i.e., a convex quadratic objective function with linear constraints. Convex programming models have fair properties: robustness, global optimality of any local optimum, and solvable in polynomial time. Quadratic programming models can be solved very efficiently by adaptations of standard methods of linear programming, such as simplex algorithm [9] and the barrier type algorithms [10]. Thus, the model can be solved with small elapsed times by standard optimisers whenever the model dimensions are not oversized. For 5 second regulation cycles, elapsed times are small enough using standard algorithms and optimisers.

4 Simulation results

A prototype of the regulator has been implemented, as well as a traffic simulator of loop lines, in order to test the performance of this regulation approach. The simulator is based on fixed-increment time advance, using a step of 1 s. The main characteristics of the simulated loop line (runtimes, stop times, signalling systems, the characteristics of the ATO equipment, etc.) are similar to the lines in Metro de Madrid. The target system has 48 platforms and 24 trains. The parameters that characterise the line are considered equal for every platform, that is: the stop times are 20 s, the nominal runtimes are 100 s and the minimum departure-arrival intervals are 130 s. With these parameters, the nominal headway is 240 s. For this study, the nominal headway has been chosen far from the minimum one, in order to have enough regulation margins to illustrate the comparison with different simulation results. The proportional constant α of the passenger model has been considered 0.02.

In addition, it is possible to recover 5 s during stop time at every platform and 15 s during runtime (flat out). On the other hand, the slowest runtime that can be obtained (train coasting) adds 20 s to the nominal value.

The control cycle has been adjusted to 5 seconds, and therefore the control actions are computed at this rate. When a train is at a station, its stop time control action is sent to the train every 5 s until its departure. However, the runtime control command is sent only once when the train departs and it is not possible to send it again between platforms. This behaviour reflects the technical characteristics of Metro de Madrid ATO equipment, with communications only at train departure.

The selection of the weights in the cost function (eq. 14) tries to reflect the performance criteria in Metro de Madrid when the line is operated according to a published headway. Control actions that increase stop times and runtimes have been equally weighted ($ap=bp=1$). Negative control action weights (those that recover time) are set to $an=bn=0.01$. If energy consumption were considered, a greater value of the weight an could be chosen. However, it has not been considered, because in most lines during peak hours the main goal is to maintain commercial speed. The timetable weight and headway weight are modified in order to show their influence on the regulation transient.

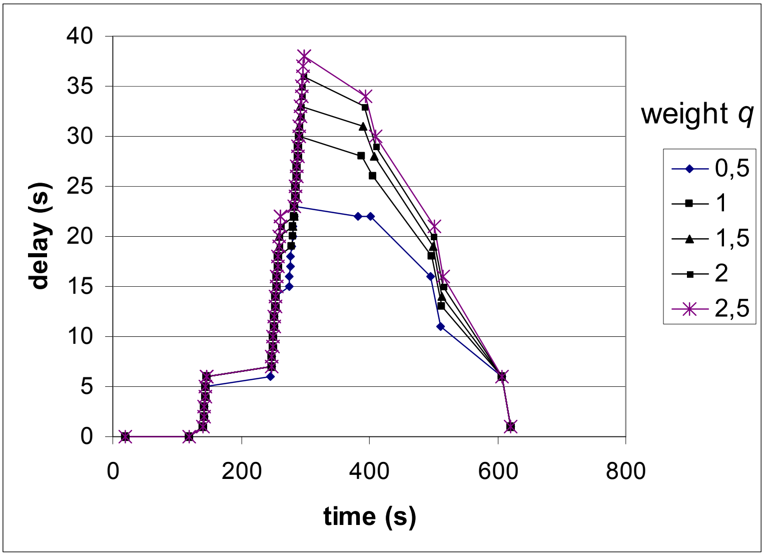

Figure 2.

Figure 2, shows the delay evolution of the train that precedes the initially delayed one (initial delay of 60 s). As expected, when the initial train is delayed, the train ahead is first held down to reduce the headway, and afterwards it is forced to speed up towards nominal operation. The timetable weight p is 0,2 and the headway weight q is varied from 0,5 up to 2,5. A greater value of q increases de timetable delay, in order to decrease the headway deviations during the transient. The adjustment of this weight depends on the operation criteria.

In order to study the stability limits of the controller, random delays have been introduced in the simulated departures. These perturbations have been modelled by means of a normal distribution function. The average of the distribution is located at 0 s but the negative values are bounded at -5s, to observe the minimum dwell time constraint.

Figure 3 shows the average delay evolution of the line, given a distribution function of random delays with standard deviation σ= 25, and a headway weight q=1. Simulation results show that the stability of the line is improved when the timetable weight is increased because this weight enhances the recovery of timetable delays. In this case, when p takes values smaller that 0,2 the system becomes unstable.

Figure 4 shows the average delay evolution of the line, once the timetable weight has been adjusted to p=0,2 and the headway weight to q=1. The evolution is analysed modifying the standard deviation of the disturbances from σ=5 to 50. When σ is greater than 25 the system becomes unstable.

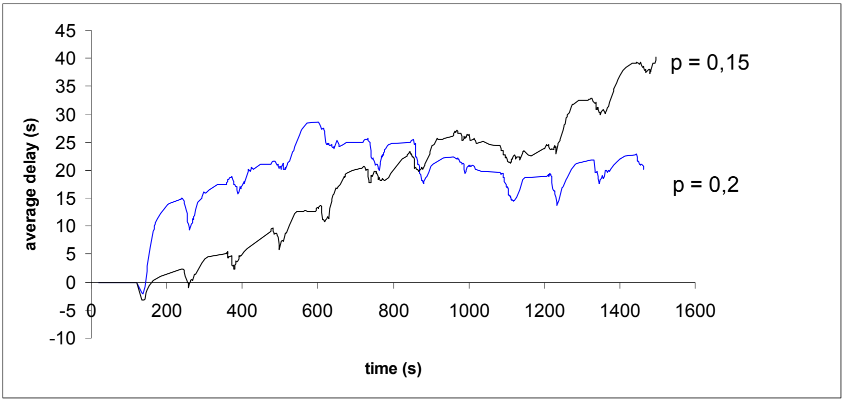

Figure 3.

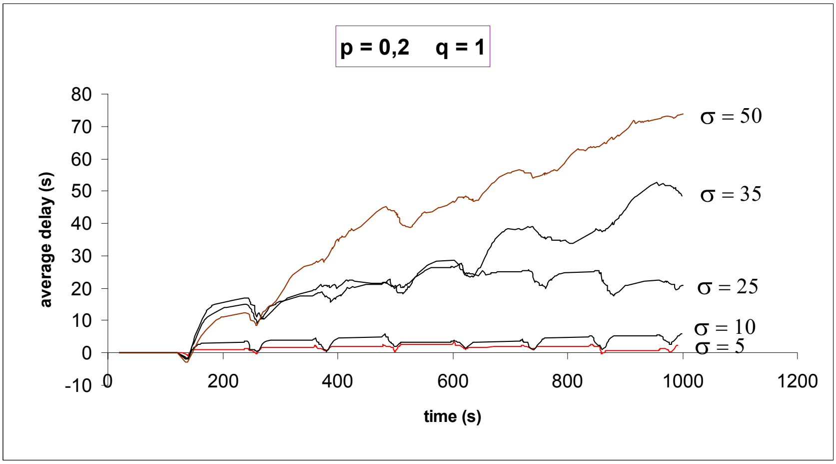

Figure 4.

5 Conclusions

A predictive method for metro loop lines regulation has been developed and tested by simulation. It is based on a traffic model that includes the main constraints (minimum interval and bounds of control actions), and the typical operation criterion to avoid the actuation of signalling systems between platforms. The predictive optimisation model, that minimises a cost function built from the 'system' point of view, is a convex quadratic programming model. The advantage of this approach is that it allows managing the main operation constraints and it can be solved very efficiently in real time.

Positive control actions (that degrade commercial speed) and negative control actions (that enhance speed and may increase energy consumption), as well as stop times and runtime control actions, are separately considered. This contributes to a realistic model that is more suitable to the operation quality indicators that will be measured in the line.

Simulation results have been obtained, and the effects of the weight adjustment on the delays during the transient and on the stability of the line have been displayed. In conclusion, the proposed regulation method is suitable to be implemented in traffic control centres, due to its performance, computational efficiency, robustness, and easy parameter-tuning according to the regulation criteria.

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